12.5

What Makes You So Special?

Special Products

LEARNING GOALS

In this lesson, you will:

- Identify and factor the difference of two squares.
- Identify and factor perfect square trinomials.
- Solve quadratic equations and functions using factoring.
- Identify and factor the difference of two cubes.
- Identify and factor the sum of cubes.

KEY TERMS

- difference of two squares
- perfect square trinomial
- difference of two cubes
- sum of two cubes

There are a number of rare elements on Earth. Precious gems are relatively rare, which is why they're so valuable.

Some blood types are rare too. The O blood type is the most common, with about 37% of the population having it. The least common is the blood type AB, with only about 4% of the population having it.

People with rare blood types are strongly encouraged to donate blood if they can, since it is more difficult to find rare blood types in cases of emergency.

What is your blood type?

PROBLEM 1 Special Products

1. Multiply the binomials.

   a. $(x - 4)(x + 4) =$ _____
   $(x + 4)(x + 4) =$ _____
   $(x - 4)(x - 4) =$ _____

   b. $(x - 3)(x + 3) =$ _____
   $(x + 3)(x + 3) =$ _____
   $(x - 3)(x - 3) =$ _____

   c. $(3x - 1)(3x + 1) =$ _____
   $(3x + 1)(3x + 1) =$ _____
   $(3x - 1)(3x - 1) =$ _____

   d. $(2x - 1)(2x + 1) =$ _____
   $(2x + 1)(2x + 1) =$ _____
   $(2x - 1)(2x - 1) =$ _____

2. What patterns do you notice between the factors and the products?

3. Multiply these binomials.
   $(ax - b)(ax + b) =$ _____
   $(ax + b)(ax + b) =$ _____
   $(ax - b)(ax - b) =$ _____

12

In Questions 1 and 3, you should have observed a few special products. The first type of special product is called the *difference of two squares*. The **difference of two squares** is an expression in the form $a^2 - b^2$ that can be factored as $(a + b)(a - b)$.

The second type of special product is called a *perfect square trinomial*. A **perfect square trinomial** is an expression in the form $a^2 + 2ab + b^2$ or in the form $a^2 - 2ab + b^2$. A perfect square trinomial can be written as the square of a binomial.

4. Identify the expressions in Questions 1 and 3 that are examples of the difference of two squares. Write both the unfactored and factored forms of each expression.

5. Identify the expressions in Questions 1 and 3 that are examples of perfect square trinomials. Write both the unfactored and factored forms of each expression.

   a. Of the form $ax^2 + 2ab + b^2$:

   b. Of the form $ax^2 - 2ab + b^2$:

6. Tizeh says that he can factor the *sum* of two squares in the same way as he factors the difference of two squares. It's just that addition will be used in both binomials. His work is shown.

Tizeh

$x^2 + 16$

$(x + 4)(x + 4)$

Cheyanne disagrees and says that to factor the sum of two squares, you should use subtraction in each binomial. Her work is shown.

Cheyanne

$x^2 + 16$

$(x - 4)(x - 4)$

Who is correct? Explain your reasoning.

12

7. Complete the table to represent each difference of squares.

a	b	$a^2 - b^2$	Factored Form
x	2		
		$4x^2 - 9$	
			$(x^2 + 4)(x^2 - 4)$
x^2	y^2		

a. What does the information in this table show?

b. Can any of the expressions in factored form be factored further? If so, factor them further.

8. Factor each polynomial, if possible.

a. $x^2 + 10x + 25$

b. $4x^2 + 20x + 25$

c. $x^2 - 24x + 144$

d. $36x^2 - 36x + 9$

e. $x^2 + 25$

f. $16x^4 - 1$

12

9. Calculate the roots of each quadratic equation.

a. $x^2 - 12x + 36 = 0$

b. $9x^2 - 25 = 0$

10. Calculate the zeros of each function.

a. $f(x) = 25x^2 + 20x + 4$

b. $f(x) = 9x^2 + 1$

c. $f(x) = 9 - 24x + 16x^2$

d. $f(x) = \frac{1}{4}x^2 - 1$

PROBLEM 2 Are Cubes Perfect Too?

In Problem 1, you dealt with special products that had degrees of 2. There are also special products with degrees of 3.

1. Use a multiplication table to determine $(x - 2)(x^2 + 2x + 4)$.

12

2. Use a multiplication table to determine $(x - y)(x^2 + xy + y^2)$.

3. Analyze your solution in Question 2.

   a. What happened to the original terms x and y?

   b. Does the solution in Question 2 follow the same pattern as the solution in Question 1? Explain your reasoning.

Each expression and product in Questions 1 and 2 represents the *difference of two cubes*. The **difference of two cubes** is an expression in the form $a^3 - b^3$ that can be factored as $(a - b)(a^2 + ab + b^2)$.

4. Each part of the factored form is related to the cube root of each term in the original expression. Identify each part of the factored form as it relates to the cube root of one of the original terms.

$$a^3 - b^3 - (a - b)(a^2 + ab + b^2)$$

5. Using the parts you just identified, explain the formula for the difference of two cubes.

12

6. Factor the difference of the two cubes: $x^3 - 27$.

Let's consider the products of two more polynomials in factored form.

7. Use multiplication tables to determine each product.

a. $(x + 5)(x^2 - 5x + 25)$

b. $(x + y)(x^2 - xy + y^2)$

Each product represents the *sum of two cubes*. Previously you determined that you cannot factor the sum of two squares. Based on your products in Question 7, you can factor the sum of two cubes. The **sum of two cubes** is an expression in the form $a^3 + b^3$ that can be factored as $(a + b)(a^2 - ab + b^2)$.

12

8. Factor the sum of the two cubes: $u^3 + 8$.

9. Analyze the expression Emilio factored.

Emilio

$64x^3 + 125$
$(4x)^3 + (5)^3$
$(4x + 5)(4x^2 - (4x)(5) + 5^2)$
$(4x + 5)(4x^2 - 20x + 25)$

Explain to Emilio what he did wrong and correctly write the expression in factored form.

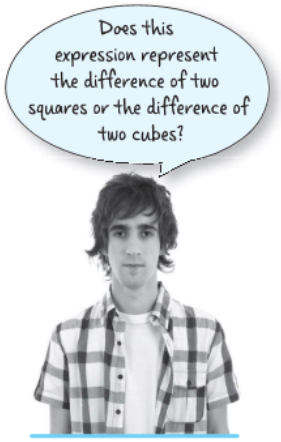
10. Sophie factored the expression shown.

Sophie

$250x^4 + 128x$
$2(125x^3 + 64)$
$2(5x + 4)((5x)^2 - (5x)(4) + 4^2)$
$2(5x + 4)(25x^2 - 20x + 16)$

12

Explain Sophie's mistake and correctly write the expression in factored form.



11. Completely factor the expression $x^6 - y^6$.

Talk the Talk

1. Complete the table to define each special product.

	Formula	Factors	Definition	Example
Perfect Square Trinomial				
Difference of squares				
Sum of Cubes				
Difference of Cubes				

2. Factor each expression.

a. $2x^2 + 18$

b. $9x^6 - y^3$

c. $2x^3 - 16$

d. $125a^3 + 27$

12

Be prepared to share your solutions and methods.